ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD

(Department of Computer Science)

WARNING

1. **PLAGIARISM OR HIRING OF GHOST WRITER(S) FOR SOLVING THE ASSIGNMENT(S) WILL DEBAR THE STUDENT FROM AWARD OF DEGREE/CERTIFICATE, IF FOUND AT ANY STAGE.**
2. **SUBMITTING ASSIGNMENTS BORROWED OR STOLEN FROM OTHER(S) AS ONE'S OWN WILL BE PENALIZED AS DEFINED IN "AIOU PLAGIARISM POLICY".**

Course: Analysis & Design of Algorithms (3466) **Semester: Autumn, 2012**
Level: BS (CS) **Total Marks: 100**
Pass Marks: 50

ASSIGNMENT-1

Units: (1 – 4)

Note: All questions are compulsory and carry equal marks.

Q. 1 a) Prove that the running time of an algorithm is $\theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$.

b) Let f(n) and g(n) be asymptotically positive functions. Prove or disprove each of the following conjectures;

a. $f(n) = \theta(f(n/2))$
b. $f(n) = O((f(n))^2)$
c. $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$

Q. 2 a) Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for sufficiently small n. make your bounds as tight as possible, and justify your answers.

a. $T(n) = 3T(n/3+5) + n/2$
b. $T(n) = 3T(n/2) + n\lg n$
c. $T(n) = T(n-1) + 1/n$

b) Prove that $Pr\{A \mid B\} + Pr\{\overline{A} \mid B\} = 1$.

Q. 3 a) Give examples of relations that are;

a. Reflexive and symmetric but not transitive
b. Reflexive and transitive but not symmetric
c. Symmetric and transitive but not reflexive

b) Let A and B be finite sets, and f : A −> B be a function. Show that:
   a. If f is injective, then $|A| \leq |B|$
   b. If f is surjective, then $|A| \geq |B|$

c) Show that any connected, undirected graph G = (V, E) satisfies $|E| \geq |V| - 1$.

Q. 4 a) Illustrate the operation of Heap sort on the array A = [5, 13, 2, 25, 7, 17, 20, 8, 4].

b) What is the running time of heap sort on an array A of length n that is already sorted in increasing order? What about decreasing order?

c) Show that the running time of Quick sort is $\theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

Q. 5 a) Illustrate the operation of Counting sort on the array A = [6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2].

b) What is the worst-case running time for the bucket-sort algorithm? What simple change to the algorithm preserves its linear expected running time and makes its worst-case running time O(n*lg*n)?

ASSIGNMENT-2

Units: (5 – 8)

Total Marks: 100 **Pass Marks: 50**

Note: All questions are compulsory and carry equal marks.

Q. 1 a) Demonstrate the insertion of the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be h(k) = k mod 9.

b) For the set of keys {1, 4, 5, 10, 16, 17, 21}, draw binary search trees of height 2, 3, 4, 5, and 6.

Q. 2 a) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <5, 10, 3, 12, 5, 50, 6>.

b) Determine an LCS of < 1, 0, 0, 1, 0, 1, 0, 1> and < 0, 1, 0, 1, 1, 0, 1, 1, 0>.

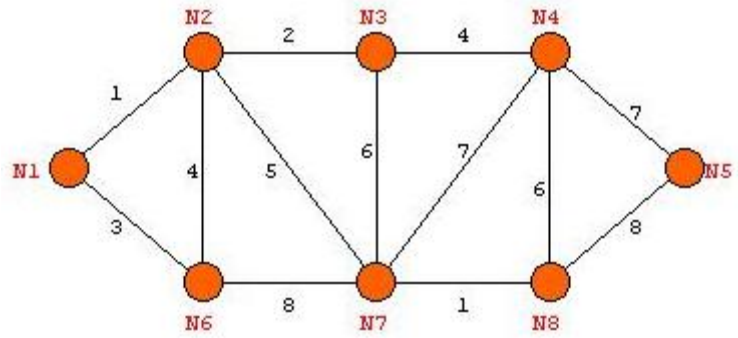
Q. 3 a) Prove that the fractional knapsack problem has the greedy-choice property.

b) What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
a : 1 b : 1 c : 2 d : 3 e : 5 f : 8 g : 13 h : 21

Q. 4 Execute the following algorithms for the given graph. Analyze the difference between the order of nodes or edges visited for the two algorithms.

a) Prim's algorithm

b) Kruskal's algorithm



Q. 5 Give and explain each step with graph example for the trace of following graph traversal algorithms.

a) Breadth first search

b) Depth first search

Analysis and Design of Algorithm (3466) Credit Hours: 3(3+0)

Recommended Book:

Introduction to Algorithms by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest

Course Outlines:

Unit No.1: **Introduction**

Introduction to Algorithm Analysis and Design

Growth of Functions, Summations Formulas and Properties

Unit No.2: **Recurrences and Sets**

Substitution, Iteration and Master Methods

Sets, Relations, Functions, Graph and Trees, Counting and Probability

Unit No.3: **Sorting Algorithms**

Heaps, Maintaining the Heap Property, Heap Sort algorithm,

Quick Sort, Performance and Analysis of Quick Sort

Unit No.4: **Sorting in Linear Time and Order Statistics**

Lower bounds for sorting, Counting sort, Radix and Bucket Sort, Medians and order Statistics

Unit No.5: **Elementary Data Structures**
Analysis of Stack, Queues and Linked List Algorithms, Hash Table and Functions, Binary Search Trees

Unit No.6: **Dynamic Programming**
Matrix Chain Multiplication, Longest Common Subsequence, Optimal Polygon Triangulation

Unit No.7: **Greedy Algorithms**
An activity selection problem, Huffman Codes, A Task Scheduling Problem, Amortized Analysis

Unit No.8: **Graph Algorithms**
Elementary Graph Algorithms, Breadth first search, Depth first search, Minimum Spanning Trees

Unit No.9: **Single Source Shortest Paths**
Shortest Paths and Relaxation, Dijkstra's Algorithm, The Bellman-Ford Algorithm, Introduction to NP-Completeness

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